Wireless Channel Modeling and Simulation

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The checklist

- **Week 1**
  - Overview of wireless communications
  - Physical Modelling of Wireless Channels
  - Path loss Doppler’s effect Multipath fading shadow fading
  - Time and frequency coherence in wireless channels
  - Using Matlab for wireless channel studies: Basics
  - End of week assignment/exercise

- **Week 2**
  - Using complex number theory (phasors) for channel modelling
  - Statistical channel models
  - Multipath fading
  - Using Matlab to simulate fading in wireless channels
  - End of week assignment/exercise
The checklist - Done

- Week 1
  - Overview of wireless communications
  - Physical Modelling of Wireless Channels
  - Path loss Doppler’s effect Multipath fading shadow fading
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  - Using Matlab for wireless channel studies: Basics
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- Week 2
  - Using complex number theory (phasors) for channel modelling
  - Statistical channel models
  - Multipath fading
  - Using Matlab to simulate fading in wireless channels
  - End of week assignment/exercise
This week’s focus

- Some follow-on for Doppler’s Shift in fixed reflecting wall example
- Combined assignment/exercises for two weeks
- Statistical channel models
- Using Matlab to simulate fading in wireless channels
Doppler shift - reflecting wall example

- The example was introduced in the Lab
- Assume the car is stationary

Transmit antenna

\[ \alpha \cos 2\pi f_c t \]

\[ r_0 = r(t = t_0) \]

\[ \frac{\alpha}{r_0} \cos 2\pi f_c \left( t - \frac{r_0}{c} \right) - \frac{\alpha}{2d - r_0} \cos 2\pi f_c \left( t - \frac{2d - r_0}{c} \right) \]
Doppler shift - reflecting wall example

\[ Z_r = \frac{\alpha}{r_0} \cos 2\pi f_c \left( t - \frac{r_0}{c} \right) - \frac{\alpha}{2d - r_0} \cos 2\pi f_c \left( t - \frac{2d - r_0}{c} \right) \]

- As the car moves rightwards,
  - the direct path increases in length
  - the reflected path decreases in length

\[ Z_r = \frac{\alpha}{r_0} \cos 2\pi f_c \left( t - \frac{r_0 + vt}{c} \right) - \frac{\alpha}{2d - r_0} \cos 2\pi f_c \left( t - \frac{2d - (r_0 + vt)}{c} \right) \]

\[ Z_r = \frac{\alpha}{r_0 + vt} \cos 2\pi f_c \left( \left( 1 - \frac{v}{c} \right) t - \frac{r_0}{c} \right) - \frac{\alpha}{2d - r_0 - vt} \cos 2\pi f_c \left( \left( 1 + \frac{v}{c} \right) t - \frac{2d - r_0}{c} \right) \]
Doppler shift - reflecting wall example

\[
Z_r = \frac{\alpha}{r_0 + vt} \cos 2\pi f_c \left( \left( 1 - \frac{v}{c} \right) t - \frac{r_0}{c} \right) - \frac{\alpha}{2d - r_0 - vt} \cos 2\pi f_c \left( \left( 1 + \frac{v}{c} \right) t - \frac{2d - r_0}{c} \right)
\]

- Assume the car is close to the reflecting wall,
  - Both paths are approximately same
  - We approximate the second path with a length \( r_0 + vt \)
- Combining the two cosines using trigonometric identity, we obtain another form of the received wave

\[
Z_r = \frac{2\alpha}{r_0 + vt} \sin 2\pi f_c \left( \frac{vt}{c} + \frac{r_0 - d}{c} \right) \sin 2\pi f_c \left( t - \frac{d}{c} \right)
\]
Doppler shift reflecting wall example

\[
Z_r = \frac{2\alpha}{r_0 + vt} \sin 2\pi f_c \left( \frac{vt}{c} + \frac{r_0 - d}{c} \right) \sin 2\pi f_c \left( t - \frac{d}{c} \right)
\]

- Product of two sinusoids
  - Right one has carrier frequency (say 900 MHz)
  - Left one has much lower frequency: scaled with the ratio \( v/c \)
    (50Hz for the example where velocity is 60km/hr and carrier frequency is 900 MHz)
Statistical Models

- Design and performance analysis based on statistical ensemble of channels rather than specific physical channel.

\[ h_\ell[m] \approx \sum_i a_i e^{-j2\pi f_c \tau_i} \]

- **Rayleigh** flat fading model: many small scattered paths

\[ h[m] \sim \mathcal{N}(0, \frac{1}{2}) + j\mathcal{N}(0, \frac{1}{2}) \sim \mathcal{CN}(0, 1) \]

  Complex circular symmetric Gaussian.

  Squared magnitude is exponentially distributed.

- **Rician** model: 1 line-of-sight plus scattered paths

\[ h_\ell[m] = \sqrt{\frac{\kappa}{\kappa + 1}} \sigma_\ell e^{j\theta} + \sqrt{\frac{1}{\kappa + 1}} \mathcal{CN}(0, \sigma_\ell^2) \quad \sigma_\ell^2 = 1 \]
Statistical Models – Rayleigh

- No line of sight
- Sum of “several” randomly rotated phasors: phase is uniformly distributed over the interval \((0, 2\pi)\)
- Both real and imaginary parts are
  - Independent
  - Identically distributed
  - Distribution is given by the “normal” or “Gaussian”
  - Each has variance equal to half the variance of the overall complex path gain
  
\[
h[m] \sim \mathcal{N}(0, \frac{1}{2}) + j\mathcal{N}(0, \frac{1}{2}) \sim \mathcal{CN}(0, 1)
\]
Statistical Models – Rayleigh circular symmetry

- Complex Gaussian is “circularly symmetric” implies that phase is uniformly distributed
  - i.e. we do not give any preference to a specific phase value
  - this is well inline with reality

\[ h[m] \sim N(0, \frac{1}{2}) + jN(0, \frac{1}{2}) \sim CN(0, 1) \]
Distribution of squared magnitude

- Squared magnitude is exponentially distributed

![Graph showing the distribution of squared magnitude value with a peak at approximately 1.5 and decreasing towards 0.]
Rician Distribution Model

\[ h_\ell[m] = \sqrt{\frac{\kappa}{\kappa + 1}} \sigma_\ell e^{j\theta} + \sqrt{\frac{1}{\kappa + 1}} \mathcal{CN}(0, \sigma^2_\ell) \]
\[ \sigma^2_\ell = 1 \]

- Power in direct Line of Sight LoS = \( \frac{\kappa}{\kappa + 1} \)
- Power in scattered paths = \( \frac{1}{\kappa + 1} \)
- Total Power = 1
- Ratio of Power in LoS and Scattered paths = \( \kappa \)
- Extreme cases
  - \( \kappa \) tends to zero implies Rayleigh fading
  - \( \kappa \) tends to infinity implies deterministic pure LoS path gain
Correlation over time

- Specified by autocorrelation function and power spectral density of fading process.
- Example: Clarke’s (or Jake’s) model.
Additive Gaussian Noise - revisited

- Complete baseband-equivalent channel model:
  \[ y[m] = \sum_{\ell} h_{\ell}[m] x[m - \ell] + w[m] \]
  \[ w[m] \sim \mathcal{CN}(0, N_0) \]

- Special case: flat fading:
  \[ y[m] = h[m] x[m] + w[m] \]

- Will use this throughout the course/simulations.
Exercise A (Slide 13 Lec 1)

- To keep error probability same, how shall the inputs be adjusted?

Hint 1
Error proportional to “Signal power/Noise Variance”

Hint 2
Noise Variance = $\sigma^2$

Hint 3
Signal Power proportional to “Square of input value”
Exercise A (Slide 13 Lec 1)

- Given that for standard deviation 1 an input of $\sqrt{2}$ meets our target for tolerable probability of error,
- Find the appropriate inputs to achieve same error probability when
  - Standard deviation is 2
  - Standard deviation is 0.5

Level = Easy
Exercise B (Slide 26 Lec 1)

- Use the Euler’s Formula to prove that
  \[ a \cos \theta + b \sin \theta = \sqrt{a^2 + b^2} \cos \left( \theta - \arctan \frac{b}{a} \right) \]

- Euler’s formula gives
  \[ a \cos \theta = a \frac{e^{j\theta} + e^{-j\theta}}{2} \]

- Also we can find
  \[ b \sin \theta = ? \]

- Use Euler’s formula, manipulate/re-arrange and then reuse the Euler’s formula

Level = medium
Exercise C (Calculating the Doppler’s Shift)

- Consider the equation (slide 6 for reflecting wall example)
  \[ Z_r = \frac{\alpha}{r_0} \cos 2\pi f_c \left( t - \frac{r_0 + vt}{c} \right) - \frac{\alpha}{2d - r_0} \cos 2\pi f_c \left( t - \frac{2d - (r_0 + vt)}{c} \right) \]

- Identify the time dependent delay of each path
  \[ \tau_1(t), \tau_2(t) \]

- Find the rate at which they change (derivative of the time dependent delay w.r.t. time)
  \[ \tau_1', \tau_2' \]

- Verify that the shift in effective received frequency for each path is given as
  \[ f_c \tau_1', f_c \tau_2' \]

Level = Medium
Exercise C (Calculating the Doppler’s Shift)

- Find the Doppler spread (the difference between the frequency shift for the two paths in the example) for the following parameters

  - $v = 60 \text{ km/hr}$
  - $f_c = 900 \text{ MHz}$
  - $c = 300,000,000 \text{ m/s}$

Level = Medium
Exercise D

- Assume you have a tool in Matlab that generates required number of real “random” samples from Gaussian distribution – zero mean and unity variance.

- How will you generate a sample with – non-zero mean, say 1/10, and variance = 1/2? (we require this to generate complex Rician fading)

Level = Easy
Exercise E: Rotating phasors example

- We will attempt to extend our Lab code to simulate a scenario where the car is moving with a specific velocity.
- Taking snapshots at regular intervals and plotting the phasor notation we will create an animation to help us visualise the fading due to movement of the receiver (environment stationary).
- For the time being we will ignore the distance dependent path loss.

Level = Hard
Thank You

Questions and Discussions